NAG Toolbox for MATLAB

f07ce

1 Purpose

f07ce computes the solution to a real system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$, where A is an n by n tridiagonal matrix and X and B are n by r matrices, using the LU factorization returned by f07cd.

2 Syntax

```
[b, info] = f07ce(trans, dl, d, du, du2, ipiv, b, 'n', n, 'nrhs_p',
nrhs_p)
```

3 Description

f07ce should be preceded by a call to f07cd, which uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element in each column, and U is an upper triangular band matrix, with two superdiagonals. f07ce then utilizes the factorization to solve the required equations.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

5 Parameters

5.1 Compulsory Input Parameters

1: **trans – string**

Specifies the equations to be solved as follows:

trans = 'N'

Solve $AX = B$ for X.

trans = 'T' or 'C'

Solve $A^{\mathrm{T}}X = B$ for X.

Constraint: **trans** = 'N', 'T' or 'C'.

2: **dl**$(*)$ **– double array**

Note: the dimension of the array **dl** must be at least $\max(1, \mathbf{n} - 1)$.

Must contain the $(n - 1)$ multipliers that define the matrix L of the LU factorization of A.

3: **d**$(*)$ **– double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

Must contain the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

4: **du**$(*)$ **– double array**

Note: the dimension of the array **du** must be at least $\max(1, \mathbf{n} - 1)$.

Must contain the $(n-1)$ elements of the first superdiagonal of U.

5: **du2**$(*)$ **– double array**

Note: the dimension of the array **du2** must be at least $\max(1, \mathbf{n} - 2)$.

Must contain the $(n-2)$ elements of the second superdiagonal of U.

6: **ipiv**$(*)$ **– int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Must contain the n pivot indices that define the permutation matrix P. At the ith step, row i of the matrix was interchanged with row **ipiv**(i), and **ipiv**(i) must always be either i or $(i+1)$, **ipiv**$(i) = i$ indicating that a row interchange was not performed.

7: **b**(**ldb**,$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r matrix of right-hand sides B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **d** The dimension of the array **ipiv**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldb

5.4 Output Parameters

1: **b**(**ldb**,$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **trans**, 2: **n**, 3: **nrhs_p**, 4: **dl**, 5: **d**, 6: **du**, 7: **du2**, 8: **ipiv**, 9: **b**, 10: **ldb**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1\|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

Following the use of this function f07cg can be used to estimate the condition number of A and f07ch can be used to obtain approximate error bounds.

8 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ or $A^{\mathrm{T}}X = B$ is proportional to $n \times r$.

The complex analogue of this function is f07cs.

9 Example

```
trans = 'No transpose';
dl = [0.8823529411764706;
     0.01960784313725495;
     0.1400560224089636;
     -0.01479925303454714];
d = [3.4;
     3.6;
     7;
     -6;
     -1.015373482726424];
du = [2.3;
     -5;
     -0.9;
     7.1];
du2 = [-1;
     1.9;
     8];
ipiv = [int32(2);
     int32(3);
     int32(4);
```

```
     int32(5);
     int32(5)];
b = [2.7, 6.6;
     -0.5, 10.8;
     2.6, -3.2;
     0.6, -11.2;
     2.7, 19.1];
[bOut, info] = f07ce(trans, dl, d, du, du2, ipiv, b)
```

```
bOut =
   -4.0000    5.0000
    7.0000   -4.0000
    3.0000   -3.0000
   -4.0000   -2.0000
   -3.0000    1.0000
info =
           0
```